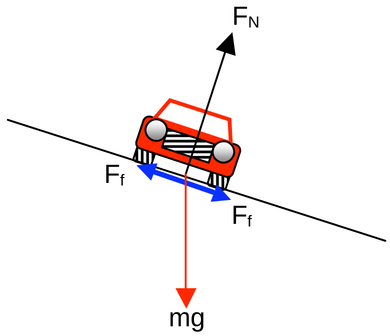
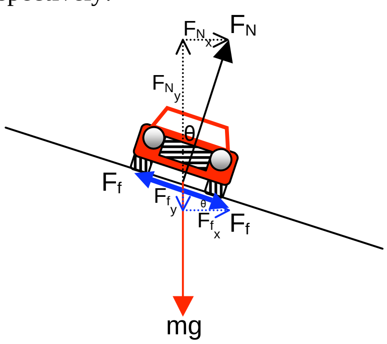
Cross Fire:

Forces at work on a car moving through a banked curve

"The only forces at work are those done by the road surface on the car. Any reference to centrifugal 'force' is so much pseudo-science drivel: it's inertia."



Since the car isn't slipping the net force, and hence net acceleration, on it must be zero. This means that all forces in the vertical and horizontal components must be zero. The sources of acceleration are the weight of the car, the normal force of the road, and the friction; the latter will act either upslope (out of the turn) or downslope (into the turn), depending on whether the car is going too slow or too fast around the turn, respectively.



Take the case of the car rounding the turn too fast. If the forces up and into the turn are positive, then the sum of all horizontal forces must equal the centripetal acceleration:

$$F_{N\,x} + F_{f\,x} = F_c$$

$$F_N \sin\theta + F_f \cos\theta = F_c$$

And the sum of all vertical forces must equal zero:

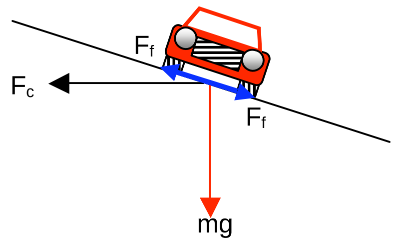
$$F_{N\,y} - F_{f\,y} - mg = 0$$

$$F_N \cos\theta - F_f \sin\theta - mg = 0$$

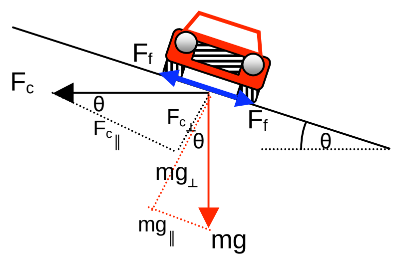
Since friction is in turn determined by the net force normal to the surface, i.e.

$$F_f = \mu F_N$$

"The driver (and the car) is just as valid a frame of reference as is the stationary road. Any force that the driver feels can be treated as real."



Since the car isn't slipping the net force, and hence net acceleration, on it must be zero. This means that all forces parallel to the road surface must be zero. The sources of acceleration are the weight of the car, the centrifugal force it feels, and the friction; the latter will act either upslope (out of the turn) or downslope (into the turn), depending on whether the car is going too slow or too fast around the turn, respectively.



Take the case of the car rounding the turn too fast. If the forces out of the turn are positive, then the sum of all parallel forces must equal zero:

$$F_{c\parallel} - mg_{\parallel} - F_f = 0$$

$$F_c \cos\theta - mg \sin\theta - F_f = 0$$

Since friction is in turn determined by the net force normal to the surface, i.e.

$$F_f = \mu F_N$$

$$F_f = \mu\left(F_{c_\perp} + mg_\perp\right)$$

$$F_f = \mu\left(F \sin\theta + mg \cos\theta\right)$$

The above equations for net force become:

$$F_N \sin\theta + \mu F_N \cos\theta = F_c$$

or

$$F_N(\sin\theta + \mu\cos\theta) = m\frac{v^2}{r}$$

and

$$F_N \cos\theta - \mu F_N \sin\theta - mg = 0$$

or

$$F_N(\cos\theta - \mu\sin\theta) = mg$$

respectively. F_N and m can be eliminated from the two equations by dividing the first by the second:

$$\frac{v^2}{rg} = \frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$v^2 = rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

For the reverse case, with the vehicle traveling too slowly, the direction of friction is reversed:

$$v^2 = rg\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$$

Which, as you can see, is totally different from what the pseudo-physics side…what-the-hey?…

Then the previous equation for net parallel force becomes:

$$F_c\cos\theta - mg\sin\theta - \mu(F_c\sin\theta + mg\cos\theta) = 0$$

$$F_c(\cos\theta - \mu\sin\theta) - mg(\sin\theta + \mu\cos\theta) = 0$$

Given that centripetal, and therefore, centrifugal, force = mv^2/r,

$$mv^2/r(\cos\theta - \mu\sin\theta) - mg(\sin\theta + \mu\cos\theta) = 0$$

$$v^2/r(\cos\theta - \mu\sin\theta) - g(\sin\theta + \mu\cos\theta) = 0$$

$$v^2/r(\cos\theta - \mu\sin\theta) = g(\sin\theta + \mu\cos\theta)$$

$$v^2(\cos\theta - \mu\sin\theta) = rg(\sin\theta + \mu\cos\theta)$$

$$v^2 = rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

For the reverse case, with the vehicle traveling too slowly, the direction of friction is reversed:

$$v^2 = rg\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$$

…which is exactly what the real force version determined.

Furthermore, the formulas can be further simplified to:

For the maximum speed possible:

$$v_{max}^2 = rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$v_{max}^2 = rg\frac{(\tan\theta + \mu)}{(1 - \mu\tan\theta)}$$

and for the minimum speed allowed:

$$v_{min}^2 = rg\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$$

$$v_{min}^2 = rg\frac{(\tan\theta - \mu)}{(1 + \mu\tan\theta)}$$

Note that in both cases, if the curve is banked so as not to require friction, i.e. $\mu=0$, then

$$v^2 = rg(\tan\theta)$$